# *Book Notes*

Helping Students Become Powerful Mathematical Thinkers: Case Studies of Teaching for Robust Understanding
by Alan Schoenfeld, Heather Fink, Sandra Zuñiga-Ruiz, Siqi Huang, Xinyu Wei, and Brantina Chirinda
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Mathematician and educator Paul Lockhart (2002), in his famous lament, posed the following analogy: a musician has a nightmare about a world in which music education becomes mandatory; but rather than playing, listening to, and composing any music, students' musical instruction consists of memorizing circles of fifths and meticulously rehearsing music notation. There are no instruments in sight. The students are bored and uninspired. Lockhart cheekily goes on:

> Waking up in a cold sweat, the musician realizes, gratefully, that it was all just a crazy dream. "Of course!" he reassures himself. "No society would ever reduce such a beautiful and meaningful art form to something so mindless and trivial; no culture could be so cruel to its children as to deprive them of such a natural, satisfying means of human expression. How absurd!" (2)

What Lockhart's playful analogy illustrates is a commentary on the fragmented state of mathematics education. Like music, with its inherent aesthetic value and interconnected parts, the discipline of mathematics can also be thought of as an art form—full of patterns and unique ways of understanding them, the worked problems like "beautiful little poems of thought . . . sonnets of pure reason" (Lockhart, 2002, 4). Yet, in the United States students do not get enough of a chance to, figuratively speaking, listen to and compose music. Even further, the United States has one of the widest math performance distributions in the world, as measured by the difference between the tenth and ninetieth percentiles of eighth graders on the Trends in International Mathematics and Science Study (NCES, 2019)—a sobering indicator of systematic educational inequity in mathematics. In their math instruction, students may get to rehearse small, disconnected bits—such as applying a formula repeatedly—without having a chance to see the whole of the discipline, what education scholar David Perkins (2009) describes as playing the whole

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game. Many students might readily recite the quadratic formula by memory and plug in values to solve it, yet few would be able to coherently explain what solving a quadratic represents conceptually. Trina Wilkerson, the former president of the National Council of Teachers of Mathematics similarly noticed this disjointedness in mathematics learning experiences, urging: “If we could view mathematics from its connectedness, then students as well as teachers would understand more deeply” (Sparks, 2023). What would it look like for students—all students—to be given the opportunity to develop integrated, deep understandings in mathematics?

In their book *Helping Students Become Powerful Mathematical Thinkers: Case Studies of Teaching for Robust Understanding,* Alan Schoenfeld and colleagues offer the comprehensive Teaching for Robust Understanding (TRU) framework to describe the core attributes of learning environments that support students in building rich and coherent understandings in mathematics. By “robust” they mean supporting students in becoming “resourceful disciplinary thinkers” (3) who are knowledgeable in the domain and flexible when it comes to understanding problems multiple ways. The book is organized into three parts. “Part I: The Big Ideas in Teaching and Learning” provides an introduction to the dimensions of the TRU framework, their origins, and how they fit together. “Part II: Reflecting on Images of Practice (The Case Studies)” provides a deep dive into practice. Here readers are invited into the classroom to reflect on practice with three detailed case studies, using the TRU framework as a lens. In “Part III: Conclusions and Next Steps,” the authors share takeaways from the case studies and offer concrete tools and resources for planning and implementation.

In Part I readers are introduced to the five dimensions of the TRU framework from both the practitioner and student perspectives. The dimensions are compact yet cover broad territory pertaining to math instruction and its complexities, as they are designed to provide a comprehensive “language and framework for inquiring into instruction and improving it” (6). The first dimension is concerned with the discipline of mathematics. What is the extent to which students are given the opportunity to think about and practice the “whole” of mathematics, not unlike what a mathematician or expert would do? This approach would be akin to having students learn music by playing music. The whole of the discipline involves both its content (the what) and practices (the how). By “content,” Schoenfeld and colleagues mean “the combination of disciplinary orientations, knowledge (including concepts and tools), practices, and habits of mind” (7) that involve opportunities to think mathematically, such as problem posing, problem solving, making conjectures, drawing connections, modeling, reasoning, sharing representations, and communicating mathematically.

The second dimension pertains to cognitive demand and opportunities for students to be challenged in productive ways. This “Goldilocks spot” between task challenge and feasibility has also been termed “productive struggle”

(Stein & Smith, 1988), a concept supported by the Vygotskian (1978) theory of learning as socially situated and ideally occurring within a learner's zone of proximal development. It is here where students are provided with support and with challenges beyond their comfort zone to facilitate learning. This dimension of the framework is at its core an asset-based view of the learner that acknowledges their inherent sensemaking abilities, building on them to expand knowledge structures and skills in mathematics.

The third dimension centers on equitable access and engagement. It asks the question: To what extent are students invited and supported to be actively engaged with disciplinary content and practices? As mathematics achievement has been historically tied to race/ethnicity, gender identity, socioeconomic status, language status, ability, and the intersections of these identities, this dimension invites reflection on *who* gets to participate and *how*. Building on the equity component of the framework is the fourth dimension, which centers on agency, ownership, and identity. Scholarship on mathematics education has offered critical analyses of power in the math classroom, including problematizing "the privileging of school math knowledge" (Nasir et al., 2008, 197). Down to classroom discourse practices—from student opportunities to share emergent and "informal" understandings to the use of their contributions as a valued resource for learning—students witness and internalize what gets welcomed as legitimate mathematical participation, shaping their mathematical identities and orientations to the classroom and disciplinary culture. This dimension of the framework concerns the socially and culturally mediated nature of mathematics and supports educators in complicating what it means to know and understand within the discipline.

The final component of the framework is formative assessment: What is the extent to which student thinking is monitored to inform instruction during the learning process? Making learning visible through artifacts of learning and discussion has a profound impact on a teacher's ability to respond to emergent student understandings. For instance, having evidence of a student's interpretation of an exponential growth model for the spread of a highly contagious disease is fundamental to a teacher's ability to design appropriate scaffolds or bridges for learning. Formative assessment enables a teacher to be accountable to both students and the discipline. Perhaps most significant is that the five dimensions of the TRU framework are designed to work in tandem. For example, promoting the active and meaningful engagement of all students (third dimension) and making their learning visible (fifth dimension) enable a teacher to provide the appropriate level of cognitive demand (second dimension).

Importantly, this book is not a prescriptive "how-to" guide; rather, it outlines a set of principles for educators to reflect on when examining math instruction. In Part II the authors offer three detailed case studies in the format of a lesson study, including a deep dive into the mathematical content of the task, the classroom context and configurations, and transcripts of episodes of interactions. They present the case studies and analyze them using the TRU framework, identifying moments of instruction to closely examine what occurred and consider what alternative options the teacher had and their potential outcomes in service of developing educator capacity for planning, observation, and reflection. Lee Shulman (1986), in his argument for meaningful teacher development programs that better accommodate teacher conceptions of process and content, identified the case method as a particularly powerful tool:

> The professional holds knowledge, not only of how—the capacity for skilled performance—but of what and why. The teacher is not only a master of procedure but also of content and rationale, and capable of explaining why something is done. The teacher is capable of reflection leading to self-knowledge, the metacognitive awareness that distinguishes draftsman from architect, bookkeeper from auditor. (13)

Part II of *Helping Students Become Powerful Mathematical Thinkers* is decidedly teacher centered, and the case study method is a particularly effective approach that honors teachers' professional judgment around the development of their instructional capacities and decision-making. From a teacher development perspective, the book's close pairing of principles with cases coheres with sound practices for teacher professional development.

The usefulness of this book for practice is solidified in Part III, where the authors provide practical tools and resources for planning and implementation. These resources serve as a launching point for any educator seeking to build their math instruction, as well as for administrators and school or system leaders seeking to improve their capacity to support math educators, mathematics instructional coaches and department chairs searching for resources for professional development, and teacher education program faculty designing math methods courses for preservice teachers. This book is particularly relevant to those serving in secondary math contexts.

With education systems still reeling from the shock of the COVID-19 pandemic and with debates about perceived learning losses reverberating throughout education discourse, some might argue for remedial mathematics programs—a return to "the basics" with simplified and rote application of disconnected algorithms. Yet such a decision would be akin to removing instruments from a music instruction program at a time when, more than ever, students need the opportunity to listen to and compose sonatas and ballads. In *Helping Students Become Powerful Mathematical Thinkers,* Schoenfeld and colleagues offer a powerful vision and guide for practitioners looking to do just that in mathematics education.

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